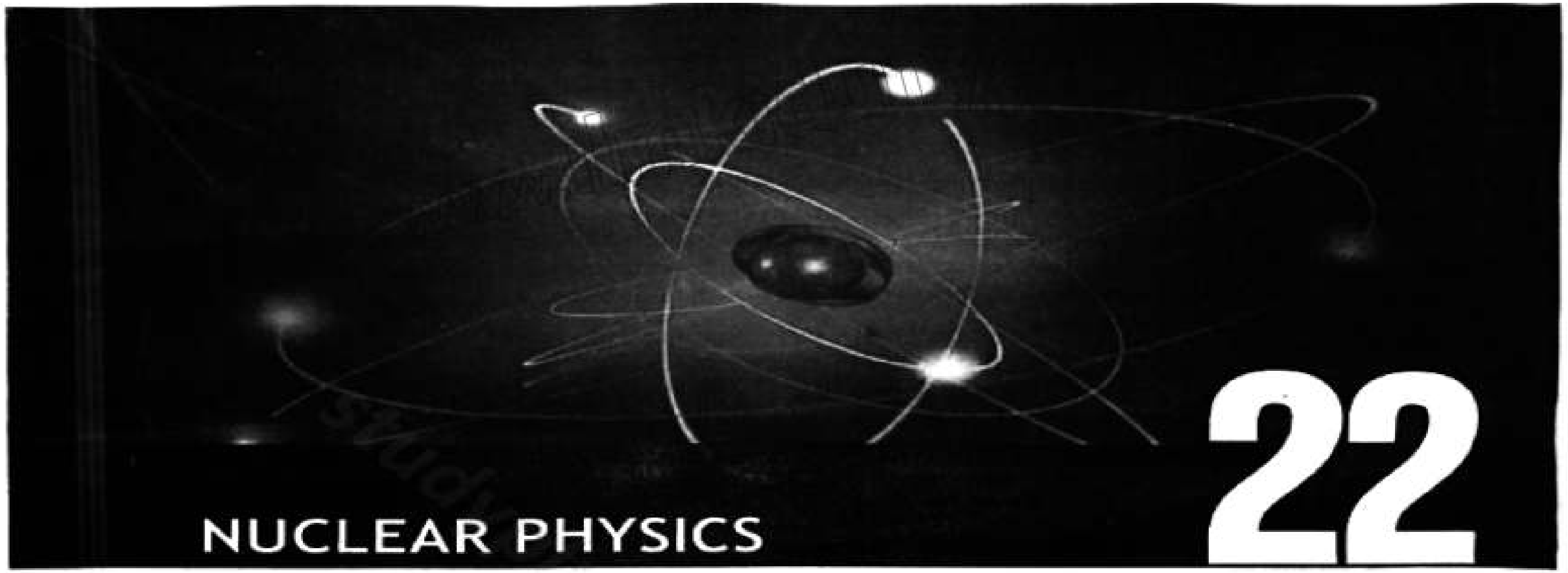


22 NUCLEAR PHYSICS

Student Learning Outcomes (SLOs)

The student will

- Recognize the equivalence between energy and mass as represented by $E = \Delta mc^2$ and state and use this equation.
- define and use the terms mass defect and binding energy.
- sketch the variation of binding energy per nucleon with nucleon number.
- Recall what is meant by nuclear fusion and nuclear fission.
- Explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission.
- Explain how the neutrons produced in fission create a chain reaction and that this is controlled in a nuclear reactor [including the action of coolant, moderators and control rods].
- calculate the energy released in nuclear reactions using $E = \Delta mc^2$.
- Explain that fluctuations in count rate provide evidence for the random nature of radioactive decay.
- explain that radioactive decay is both spontaneous and random.
- define activity and decay constant, and state and use $A = N\lambda$.
- Explain half-life with examples.
- use $= 0.693/t_{1/2}$ to solve numerical problems.
- state the exponential nature of radioactive decay.
- use the relationship $x = x_o e^{-\lambda t}$ [where x could represent activity, number of undecayed nuclei or received count rate) to solve problems analytically and graphically].
- describe the function of the principle components of water moderated power reactor [core, fuel, rods, moderator, control rods, heat exchange, safety rods and shielding]
- explain why uranium fuel needs to be enriched before use.
- compare the amount of energy released in a fission reaction with the (given) energy released in a chemical reaction.
- Explain what is a medical tracer [a substance containing radioactive nuclei that can be introduced into the body and is then absorbed by the tissue being studied].
- Explain annihilation reactions [they occur when a particle interacts with its antiparticle and that mass-energy and momentum are conserved in the process].
- Illustrate how PET scanning works [positrons emitted by the decay of the tracer annihilate when they interact with electrons in the tissue, producing a pair of gamma-ray photons traveling in opposite directions]
- calculate the energy of the gamma-ray photons emitted during the annihilation of an electron-positron pair.
- Explain that the gamma-ray photons from an annihilation event travel outside the body and can be detected [including that an image of the tracer concentration in the tissue can be created by processing the arrival times of the gamma-ray photons].

Nuclear physics is a branch of physics that deals with the study of the atomic nucleus, its properties and interactions. We studied in grade-10, that the atomic nucleus is composed of protons and neutrons, which are collectively referred as nucleons. The atomic number 'Z' of an element corresponds to the number of protons in the nucleus, while the mass number 'A' is the sum of the number of protons and neutrons in the nucleus. Number of neutrons in an atom is represented by 'N' and is called neutron number. The relation between atomic number, mass number and neutron number can be given by the following equation: $A = Z + N$.

Nuclear physics has numerous applications, including nuclear power generation, nuclear medicine, and nuclear weapons. It also plays a crucial role in the study of astrophysics, as nuclear reactions are responsible for the energy production in stars and other celestial bodies.

22.1 MASS DEFECT

If we have a polythene bag of negligible mass containing 10 balls, each ball having a mass of 100 grams, then what should be the mass of the whole basket? Definitely it would be 1000 grams (1 kg) as it is our common observation that the mass of whole is the sum of masses of the constituents. But the same is not true for the nucleus. The total mass of nucleus is less than the sum of masses of its individual nucleons. For example, the mass of carbon should be greater than 12 u, as its constituent protons and neutrons have masses greater than 1 u but it has a mass of exactly 12 u which is less than the sum of masses of six protons and six neutrons. Similarly, mass defect for binding of one proton and one neutron in case of deuterium (^2H) can be shown in Fig. 22.1. Let '$m_{nucleus}$' be the mass of nucleus, 'm_p' is the mass of proton and 'm_n' is the mass of neutron then during the formation of the nucleus 'Δm' is the difference in mass which is known as the mass defect. Mathematically the mass defect can be given as:

For Your Information

To express the masses of atoms and nuclei in nuclear physics we use 'atomic mass unit' (amu). It is defined as one twelfth of the mass of a carbon-12 atom. It can be given as:

$$1 \text{ amu} = 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

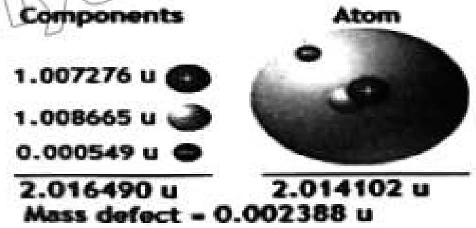


Figure 22.1: Mass defect in deuterium atom.

$$\Delta m = (m_n + m_p) - m_{nucleus}$$

The mass of deuterium is less than the sum of the masses of *one proton and one neutron*. This difference in mass of a nucleus is called as mass defect, and can be defined as:

The difference between the mass of the nucleus and sum of the masses of its constituent particles is called as mass defect.

In general, the equation for the mass defect can be given as:

$$\Delta m = [(A - Z)\, m_n + Z\, m_p] - m_{nucleus} \qquad (22.1)$$

Example 22.1. Find the mass defect in the formation of Helium-4 nucleus. If the mass of He-4 is 4.002603u, the mass of proton is 1.007276u and the mass of neutron is 1.008665u.

Given: Mass of He-4: $= m_{He} = 4.002603$u Mass of proton $= m_p = 1.007276$u

Mass of neutron = m_n = 1.008665u

To Find: Mass defect = Δm = ?

Solution: To find the mass defect we use: $\Delta m = (A - Z)\, m_n + Z\, m_p - m_{nucleus}$

As we know that for helium-4 nucleus there are two protons and two neutrons hence Z=2 and (A−Z)= 4 − 2 = 2 now putting valeues, we get:

$$\Delta m = \left[(4-2)\ (1.008665) + 2\ (1.007276) - 4.002603\right]\ u = 0.029279\ u$$

Assignment 22.1

Find the mass defect of tritium nucleus. The mass of tritium is 5.0083×10^{-27} kg.

22.2 BINDING ENERGY

The mass defect remained a mystery for some time. But soon the scientists found that in formation of the nucleus, the missing mass is converted into energy. As during the formation of nucleus, energy is found to be released and conversely to break the nucleus into its parts. An equal amount of energy is to be given for this break. The mass converted into the energy is called as binding energy and can be defined as:

The minimum energy required to break an isolated nuclei into its constituent particles is called the binding energy.

The Einstein's famous equation relates this energy and mass which can be given as:

$$E = \Delta m\, c^2 \qquad (22.2)$$

Here 'E' is the energy required to break the nucleus or the energy released during formation of the nucleus, 'Δm' is the mass defect and 'c' is the speed of light whose approximate value is $3 \times 10^8\ m\ s^{-1}$. This relation proves that the mass can be converted into energy and energy can be converted back into the mass i.e. mass and energy are inter-convertible quantities. The relation between binding energy and the mass can be shown in Fig. 22.2 (a). It is to note that the units for mass used in above equation should be in kilograms. For example, helium-4 nucleus containing two protons and two neutrons has a combined mass of 4.002603 amu while the sum of masses of two protons and two neutrons is 4.031882 amu. The mass defect can be given as:

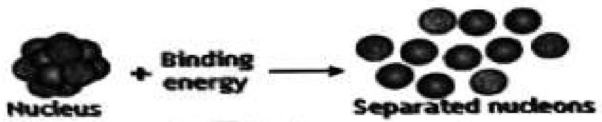


Figure 22.2 (a): Binding energy.

$$\Delta m = (4.031882 - 4.002603)\ amu = 0.029279\ amu$$

The equivalence of this mass in kg can be given as:

$$1\ amu = 1.66 \times 10^{-27} kg \qquad or \qquad 0.029279\ amu = (0.029279 \times 1.66 \times 10^{-27}) kg$$

$$\Delta m = 0.029279\ amu = (4.860314 \times 10^{-29}) kg$$

Using equation (22.3) we find:

$$E = (4.860314 \times 10^{-29} kg)\ (3 \times 10^8 ms^{-1})^2 = 4.374283 \times 10^{-12} J$$

This is the amount of energy required to break the Helium-4 nucleus. This is relatively a small amount of energy but it is enough to hold the nucleus together and give it its stability. It is often convenient in nuclear physics to express certain masses in energy units as they are simply interchangeable. According to Einstein's mass-energy equivalence relation: $E = mc^2$

For one amu:

Energy for 1 amu $= (1.66\times10^{-27}\text{kg})\ (3\times10^{8}\text{ms}^{-1})^2$

Energy for 1 amu $= 1.494\times10^{-10}$ J

As we know that: $1\text{ eV} = 1.60\times10^{-19}$ J

Hence: $\frac{1}{1.60\times10^{-19}}\text{eV} = 1\text{ J}$

Now energy equivalence for 1 amu mass in electron-volt units is:

$$1\text{ amu} = 1.494\times10^{-10}\times\frac{1}{1.60\times10^{-19}}\text{eV}$$

$$= 9.315\times10^{8}\text{ eV}$$

$$1\text{ amu} = 931.5\text{ MeV}$$

The rest mass and energy equivalent of some particles and nuclei is given in table 22.1.

Table 22.1: Masses of some particles

Particle	Mass (kg)	Mass (amu)	MeV
Electron	9.1164×10^{-31}	0.0005485	0.511
Proton	1.672×10^{-27}	1.007276	938.27
Neutron	1.675×10^{-27}	1.008665	939.57
Hydrogen	1.672×10^{-27}	1.007825	938.78
Deuterium	3.344×10^{-27}	2.014102	1876.12
Tritium	5.0083×10^{-27}	3.016049	2809.43
Helium	6.646×10^{-27}	4.002603	3728.40
Carbon-12	1.992×10^{-26}	12.000000	11177.9

Example 22.2. Find the energy required to break tritium nucleus (having mass 3.016049 u) in joules and in eV. If masses of proton and neutron are 1.007276 u and 1.008665u.

Given: Mass of tritium = M = 3.016049u   Number of protons = Z = 1

Mass of proton = m_p = 1.007276u   Number of neutrons = A−Z = N = 2

Mass of neutron = m_n = 1.008665u

To Find: Binding energy = E = ?

Solution: To find the binding energy first we find the mass defect:

$$\Delta m = (A-Z)\ m_n + Z\ m_p - M$$

Putting values, we get:

$$\Delta m = \left((2)\ 1.008665 + (1)\ 1.007276 - 3.016049\right)\ \text{amu} = 0.008557\ \text{amu}$$

To convert this value in kg:

$$\Delta m = (0.008557\times1.66\times10^{-27})\ \text{kg} = 1.420\times10^{-29}\ \text{kg}$$

Now to find the binding energy we use:

$$E = \Delta mc^2$$

$$E = (1.420\times10^{-29}\text{kg})\ (3\times10^{8}\text{ms}^{-1})^2 = 1.278\times10^{-12}\ \text{J}$$

To convert this energy into electron-volt we use:

$$E = 1.278\times10^{-12}\times\frac{1}{1.60\times10^{-19}}\text{eV} = 7.99\times10^{6}\ \text{eV}$$

Assignment 22.2

Find the binding energy of deuterium nucleus. The mass of deuterium is 3.344×10^{-27} kg.

Binding Energy per Nucleon

The binding energy of an atomic nucleus is the amount of energy required to completely separate all of its constituent nucleons (protons and neutrons). It is a measure of the strength of the nuclear force that holds the nucleus together. The binding energy is usually expressed in electron-volts or in joules. As from equation (22.2):

$$E = \Delta mc^2$$

Using the values of mass defect 'Δm' from equation (22.1) we get:

$$E = \left((A - Z)\ m_n + Z\ m_p - m_{nucleus}\right)\ c^2$$

Here 'E' is the binding energy. The absolute value of binding energy does not explain completely the stability of a nucleus. The more important thing to explain the stability of a nucleus is the binding energy per nucleon i.e. how much mass is converted by each nucleon to form a nucleus. The greater the binding energy per nucleon the greater will be the stability of the nucleus. The binding energy per nucleon also called as packing fraction (f) and can be given as:

Table 22.2: Binding energy of some nuclei in MeV

Nucleus	Binding energy	Binding energy per nucleon
^{2}H	2.23	1.12
^{4}He	28.3	7.07
^{12}C	89.1	7.4
^{56}Fe	486.24	8.68
^{206}Pb	1622	7.87
^{235}U	1786	7.6

$$f = \frac{E}{A} = \frac{\left((A - Z)\ m_n + Z\ m_p - m_{nucleus}\right)\ c^2}{A} \quad \text{------- (22.3)}$$

The packing fraction also called the binding fraction is the measure of stability of a nucleus. Therefore, the rest energy of the bound system i.e. the nucleus is less than the combined rest energy of the separated nucleons. The binding energies of some commonly used nuclei are given in the table 22.2.

Example 22.3. Find packing fraction for C-12 nucleus. If mass of C-12 is 12.000u, mass of proton is 1.007276 u and mass of neutron is 1.008665 u.

Given: Mass of carbon-12 = $m_{nucleus}$ = 12.0   Number of protons = Z = 6
Mass of proton = m_p = 1.007276u   Number of neutrons = A-Z = N = 6
Mass of neutron = m_n = 1.008665u

To Find: Packing fraction = f = ?

Solution: We have to find the binding energy as: $\Delta m = (A - Z)\ m_n + Z\ m_p - m_{nucleus}$

$$\Delta m = (6\times 1.008665 + 6\times 1.007276 - 12.000) = 0.095646\ u$$

To convert it into MeV we take:

$$E = 0.095646 \times 931.5\ MeV \quad \text{or} \quad E = 89.1\ MeV$$

For packing fraction, we use: $f = \frac{E}{A}$

Putting values, we get: $f = \frac{89.1\ MeV}{12}$ or $f = 7.4\ MeV/nucleon$

Assignment: 22.3

Find the binding energy per nucleon for Nitrogen-13 nucleus having mass 2.3258x10^{-26} kg. mass of proton is 1.007276 u and mass of neutron is 1.008665 u.

Binding Energy Curve

The variation of binding energy per nucleon with mass number can graphically be represented by a curve known as the binding energy curve. This curve shows the relationship between the number of nucleons in a nucleus (represented by the mass number A) and the binding energy per nucleon. The graph for binding energy per nucleon (in MeV) plotted against the mass number (A) as we can see from Fig. 22.2 (b), the binding energy per nucleon increases as we move from the lightest nuclei (A = 1) towards the region of medium-sized nuclei (A = 50–100), reaching a peak at around A = 56.

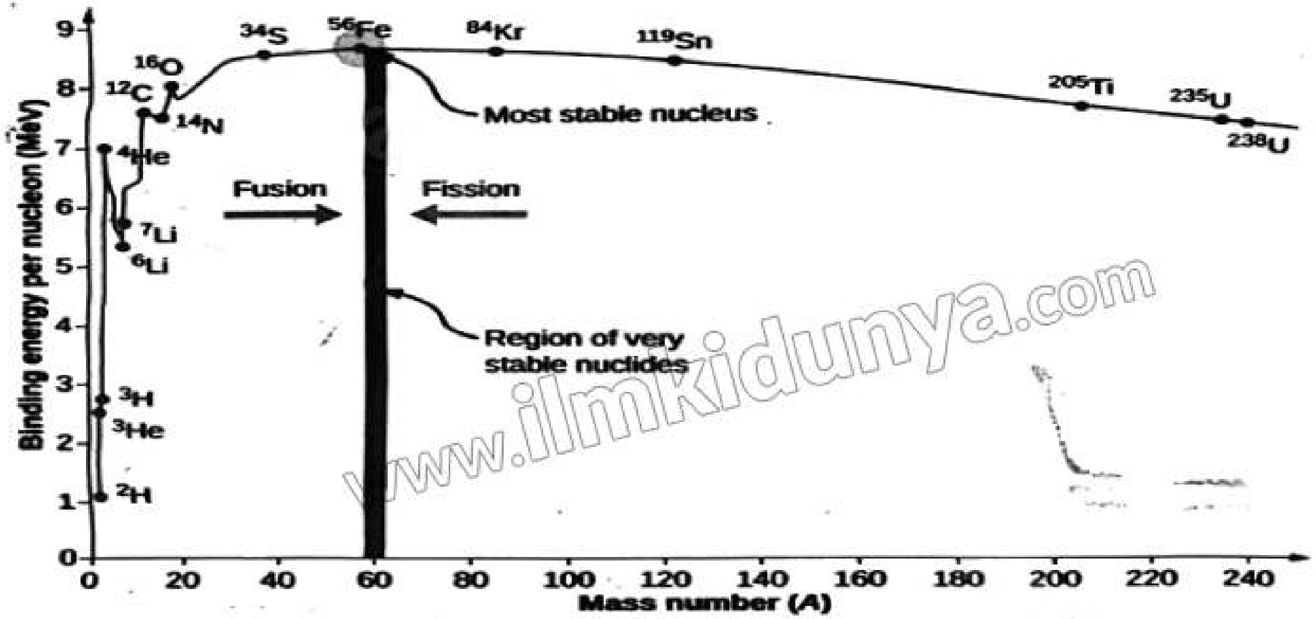


Figure 22.2 (b): Binding energy verses nucleon number

This region of medium-sized nuclei is known as the most stable nuclei or the valley of stability. Beyond the region of medium-sized nuclei, the binding energy per nucleon starts to decrease again, indicating that larger nuclei are less stable than smaller ones. This is because larger nuclei have a higher proportion of repulsive electrostatic forces between protons, which are not balanced by the attractive strong nuclear forces. The variation of binding energy per nucleon with mass number is an important factor in nuclear physics, as it helps to explain the stability and properties of atomic nuclei, and provides insight into nuclear reactions and nuclear energy production. Nuclei with a higher binding energy per nucleon are generally more stable and less likely to undergo fission. On the other hand, nuclei with a lower binding energy per nucleon, such as uranium-235, are more likely to undergo fission when they absorb a neutron, releasing energy and more neutrons, which can in turn cause further fission reactions. Fusion reactions tend to produce more stable nuclei with a higher binding energy per nucleon than the reactant nuclei. The fusion reactions release energy, as the product nucleus has a greater binding energy per nucleon than the reactant nuclei. For example, the fusion of hydrogen nuclei into helium releases a large amount of energy, as the product nucleus has a higher binding energy per nucleon than the reactant nuclei.

22.3 RADIOACTIVITY

Marie Curie studied radioactivity (nuclear radiations emission) shortly after Henri Becquerel discovered natural radioactivity. Marie Curie detected radioactivity in Uranium and Thorium using an electrometer, which showed that the air around radioactive samples became charged and conductive.

The natural emission of radiations from unstable nuclei is called radioactivity. Radioactivity is the phenomenon by which unstable atomic nuclei spontaneously decay into other nuclei, emitting radiation in the process. Spontaneous nuclear decay is the natural process by which an unstable atomic nucleus breaks down into a more stable configuration by emitting radiation. This process is spontaneous and occurs without any external influence or trigger. Radioactive decay can also result in the formation of a new element known as the daughter nuclei. Spontaneous nuclear decay is a natural process that occurs in many elements in the universe. While it can pose a significant health hazards if not properly managed, it also has many practical applications in fields such as energy production, medicine and materials sciences. There are three main types of radioactive decay alpha decay, beta decay, and gamma decay, along with these radiations some more radiations and particles are also produced like X-rays and neutrons etc.

22.3.1 Alpha Decay

Alpha decay occurs when an atomic nucleus emits an alpha particle, which is a helium nucleus consisting of two protons and two neutrons. This reduces the atomic number of the nucleus by two and the atomic mass by four. The general form of an alpha decay equation can be written as:

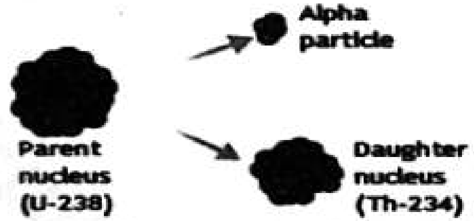


Figure 22.3: Alpha decay of U-238.

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He + Q$$

Here $^{A}_{Z}X$ is the parent nucleus which decays into $^{A-4}_{Z-2}Y$ known as daughter nucleus with emission of alpha-particle $^{4}_{2}He$ which is helium-4 nucleus and heat Q. For example, the alpha decay of uranium-238, as shown in Fig. 22.3 and can be represented as:

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$$

Here the uranium-238 nucleus (with 92 protons and 146 neutrons) decays into thorium-234 (with 90 protons and 144 neutrons) by emitting an alpha particle (with 2 protons and 2 neutrons).

22.3.2 Beta Decay

Beta decay occurs when an atomic nucleus emits a beta particle, which is an electron or a positron.

Beta decay can either increase or decrease the atomic number of the nucleus, but always conserves the total number of nucleons. Beta decays are of two types i.e. the negative beta decay and the *positive beta decay*.

Figure 22.4: Beta-minus decay.

i) Negative beta or beta-minus decay is the emission of an electron during nuclear activity. In an unstable nucleus, when a neutron decays into proton it emits an electron and an anti-neutrino. The equation for neutron decay can be given as:

$$n^0 = p^+ + {}^{0}_{-1}\beta + \bar{\nu}$$

The general equation for the nuclei emitting beta-minus decay can be given as:

$${}^{A}_{Z}X = {}_{Z+1}^{A}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

Beta-minus decay is shown in Fig. 22.4.

ii) Positive beta or beta-plus decay is the emission of positron during nuclear activity. In an unstable nucleus, when a proton decays into neutron it emits a positron and neutrino. The equation for proton decay can be given as:

$$p^+ = n^0 + {}^{0}_{+1}\beta + \nu$$

The general equation for the nuclei emitting beta-plus decay can be given as:

$${}^{A}_{Z}X = {}_{Z-1}^{A}Y + {}^{0}_{+1}\beta + \nu$$

Beta-plus decay is shown in Fig. 22.5.

For Your Information

Beta decay is used in radionuclide therapy (RNT) to treat cancer. Radioactive isotopes like lutetium-177 or yttrium-90 are used to target and destroy cancer cells. Beta decay of carbon-14 is the basis for radiocarbon dating which is a method used to determine the age of archaeological and geological samples.

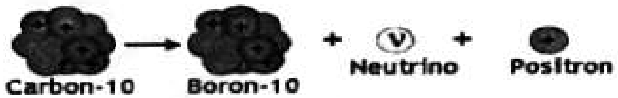


Figure 22.5: Beta-plus decay.

22.3.3 Gamma Decay

Gamma decay occurs when an atomic nucleus transitions from a higher energy state to a lower energy state, emitting a gamma ray in the process. Gamma rays, high-energy photons, are electromagnetic radiations which travel with the speed of light. Unlike alpha and beta radiation, gamma radiation does not involve the emission of massive particles. However, the emission of gamma radiation is subject to certain conservation laws that govern the properties of the nucleus. Specifically, the law of conservation of energy, momentum and

For Your Information

Gamma rays are used in radiation therapy to target and destroy cancer cells. In medical imaging *like* positron emission tomography (PET) scans, gamma rays are used to create detailed images of inside of the body. In non-destructive testing (NDT) gamma rays are employed to inspect and detect flaws in material without causing any damage.

angular momentum must be obeyed during the transition.

Even though no particles with mass are emitted during gamma decay, the emission of gamma radiation can result in the change in energy of the nucleus. For example, if a nucleus a gamma ray after undergoing alpha or beta decay it may end up in a different energy state. In such cases the nucleus may still be unstable and may continue to undergo further radioactive decay until it reaches a stable state. These laws imply that even though no particles with mass are emitted, the composition of the nucleus is certainly changed after emitting photons given by gamma decay. The gamma ray emission is shown in Fig. 22.6. The equation for gamma decay can be given as:

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

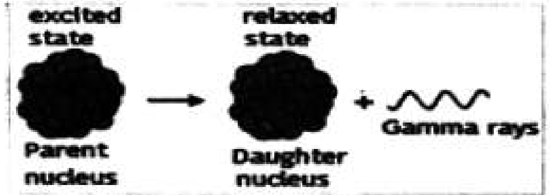


Figure 22.6: Gamma decay.

Here X^{*} represents the nucleus in an excited state. In an atom, nucleons (both protons and neutrons) have certain energy states. When they absorb energy during nuclear decay, they go up to the higher energy state and nucleus is said to be excited or in excited state. During de-excitation to the lower energy state the nucleus emits gamma radiations. Example of gamma ray emission is:

$$^{60}_{27}Co^{*} \rightarrow {}^{60}_{27}Co + \gamma$$

The properties of three types of nuclear radiations are given in the Table 22.3.

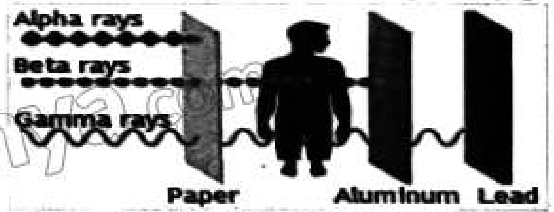


Figure 22.7: Types of radiation.

Table 22.3: Properties of nuclear radiations

Properties	Alpha	Beta	Gamma
Charge	+2	±1	0
Mass	4	Very small as compared to alpha particle	Negligible as compared to alpha particle and beta particle
Velocity	Slow (0.05 - 0.1 c)	Relatively fast (0.5 - 0.9 c)	Speed of light (c)
Penetration depth	Stopped by a few centimeters of air or a sheet of paper	Penetrate several meters of air, stopped by a few millimeters of aluminum	Penetrate several meters of air, stopped by several centimeters of lead or concrete
Typical source	Radon-222	Strontium-90	Cobalt-60

22.3.4 Spontaneous and Random Nature of Nuclear Decay

Radioactive decay is a random process i.e.; it is impossible to predict exactly when a particular unstable nucleus will decay. It may decay just on the moment you started observing it or it may not decay even for years. The decay is a natural process inherent to the unstable nucleus itself. It happens because the nucleus is seeking a more stable state. This randomness is reflected in the fluctuations observed in the count rate of radioactive emission. Count rate can be defined as:

The number of decays detected per unit time is called as count rate.

When measuring the count rate using a Geiger-Muller counter or any other counters it can be noted that the count rate varies over time even if the conditions remain constant. These fluctuations occur because each decay event is spontaneous, independent, random and unpredictable.

Each radioactive nucleus has a certain probability of decaying at any given moment but we cannot predict the exact time of decaying for any single nuclei, this property is known as "*spontaneity*". Although the individual decay events are random the average behavior of a large number of nuclei follows a predictable pattern described by the half-life of the substance. This is the reason that the count rate over longer periods becomes more stable and predictable in spite of showing fluctuations over short interval decays. These evidences provide clear evidence that decay events are not influenced by external factors and occur spontaneously. Spontaneous nuclear decay is a natural process that occurs in many elements in the universe. While it can pose a significant health hazard if not properly managed, it also has numerous practical applications in fields such as energy production, medicine, and materials science.

Example 22.4. Calculate the amount of energy 'Q' released during the following nuclear reaction. If mass of U-238 is 238.02891 u, mass of Th-234 is 234.04360 u and mass of alpha particle is 4.001506 u. $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$

Given: Mass of uranium-238 = M_U = 238.02891 u Mass of Thorium-234 = M_T = 234.04360 u

Mass of alpha particle = M_α = 4.001506 u

To Find: Energy released = Q = ?

Solution: Energy in this nuclear reaction can be calculated as:

The total mass on L.H.S is: $M_U = 238.02891$ u

The total mass on R.H.S is: $M_T + M_\alpha = 234.04360\text{ u} + 4.001506\text{ u} = 238.045106\text{ u}$

Since the difference in mass is converted into energy 'Q' then we can write as:

$$Q = M_U - (M_T + M_\alpha) = 238.02891\text{ u} - 238.045106\text{ u} = -0.016196\text{ u}$$

The negative sign shows that the energy is absorbed during the nuclear decay i.e. it is endothermic reaction. Now the amount of energy in MeV can be found as:

$$Q = -0.016196\text{ u} \times 931.5\text{ MeV/u} = 15.0866\text{ MeV}$$

Hence 15.0866 MeV energy will be released when uranium-238 nuclei decays into thorium-234 while emitting an alpha particle.

Assignment: 22.4

Find the energy 'Q' in the following reaction. If mass of C-12 is 12.000 u, mass of N-14 is 14.00307 u and mass of electron is 0.0005486 u. $^{14}_{6}C = ^{14}_{7}N + e^- + Q$

22.4 HALF LIFE AND RATE OF DECAY

The half-life ($T_{1/2}$) of a radioactive material is a characteristic property of that material and is defined as:

The time it takes for half of the radioactive nuclei to decay in a given sample is called as half-life of that element.

For example, if we have 100 nuclei at time 't = 0' and after half-life the half of the nuclei will disintegrate and half would remain intact i.e., 50 nuclei of parent element will remain. After second half-life out of these 50 nuclei 25 will disintegrate and 25 nuclei of parent element will remain similarly after third and fourth half-lives the remaining nuclei of parent element will be 12.5 and 6.25 respectively. For example, the half-life of cobalt-60 is 5.27 years and its decay with half-lives is shown in Fig. 22.8. Although after every half-life the nuclei become half of their initial value but still it takes infinite time for the disintegration of all nuclei. The rate of disintegration of nuclei follows a law known as rate law which can be defined as:

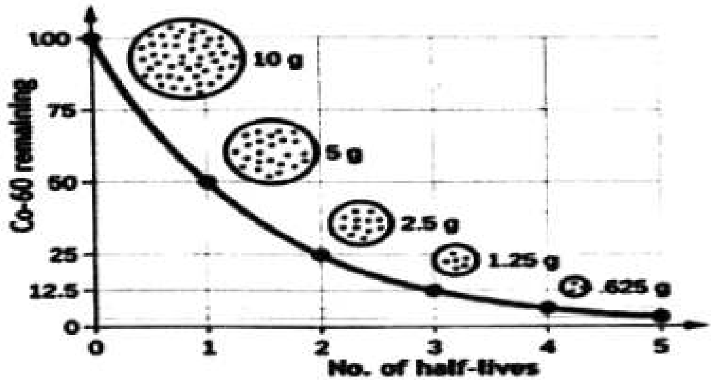


Figure 22.8: Decay of Co-60.

The rate at which a radioactive substance decays is directly proportional to the number of radioactive nuclei present at any given time, is called the rate law for nuclear decay.

Half-lives of some typical elements are given in Table 22.4. The rate of spontaneous nuclear decay is measured by the half-life. It can vary widely depending on the specific material, ranging from fractions of a second to billions of years. If 'ΔN' is the number of atoms which decay in time 'Δt' then according to decay law the time rate of spontaneous disintegration of a radioactive element is proportional to the number of nuclei 'N' present and can be given as:

Table 22.3: Half-lives of some elements

Element	Decays into	Half-life
Carbon-14	Nitrogen-14	5,730 years
Aluminum-26	Magnesium-26	740,000 years
Iodine-129	Xenon-129	17 million years
Uranium-235	Lead-207	704 million years
Uranium-238	Thorium-234	4.5 billion years
Potassium-40	Argon-40	1.3 billion years
Rubidium-87	Strontium-87	49 billion years
Plutonium-239	Uranium-235	24,110 years
Cesium-137	Barium-130	30 years
Strontium-90	Yttrium-90	28.8 years
Radon-222	Polonium-218	3.82 days

$$\frac{\Delta N}{\Delta t} \propto -N$$

Here negative sign shows that 'ΔN' is decreasing with time. Now by changing the sign of proportionality into equation a constant is multiplied as:

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Here 'λ' is a constant and is known as decay constant which is the proportionality factor in the rate law of radioactive decay. It is equal to the fraction of the radioactive of the radioactive nuclei that decay per unit time. It can be given as:

$$\lambda = \frac{1}{N} \times \frac{\Delta N}{\Delta t}$$

While the ratio '$\Delta N/\Delta t$' is called the activity in the radioactive decay process which is the number of disintegrations per unit time. It is represented by 'A' and can be given as:

$$A = \frac{\Delta N}{\Delta t}$$

$$A = -\lambda N$$

The SI unit of activity is becquerel (Bq) which can be defined as:

When a nucleus disintegrates at the rate of one disintegration per second then the activity is equal to one becquerel.

In nature, there are some radiations present everywhere (called background radiation). Hence, we are constantly exposed to radiation in daily life, but the natural levels are typically low and safe for humans and other biological objects on Earth. Harmful effects require higher radiation doses. For example, the activity of radium used in the dial of a watch (to make it glow in the dark) is 4×10^4 Bq and for radiotherapy the dose range is 4×10^{13} Bq.

A more common unit of activity is the curie which can be defined as:

When there is an activity at the rate of 3.70×10^{10} decays per second then the activity is equal to one curie.

The inter conversion of these two units of activity can be given as:

$$1\ Ci = 3.70 \times 10^{10}\ Bq$$

From the above equation, it is clear that the becquerel is very small unit as compared to the curie.

22.4.1 Exponential Nature of Nuclear Decay

Radioactive decay is exponential in nature. This means that the rate at which a radioactive substance decays is proportional to the number of un-decayed nuclei present at any given time. If 'N' be the number of nuclei at any instant which are undecayed, then

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

For very small change we can take above relation as:

$$\frac{dN}{dt} = -\lambda N$$

By the separation of variable technique, we get:

$$\frac{1}{N} dN = -\lambda\ dt$$

Taking integration on both sides: $\int \frac{1}{N} dN = -\int \lambda \, dt$

$$\ln N = -\lambda\, t + \text{constant} \quad \_\_\_\_ (i)$$

To find the value of this constant we can apply the initial conditions as: at $t = 0$, $N = N_o$

Using these values in above equation we get:

$$\ln N_o = -\lambda\,(0) + \text{constant} \qquad \text{or} \qquad \ln N_o = \text{constant}$$

Now Eq. (i) gets the form:

$$\ln N = -\lambda\, t + \ln N_o \qquad \text{or} \qquad \ln N - \ln N_o = -\lambda\, t$$

or $\ln \frac{N}{N_o} = -\lambda\, t$

Taking exponent on both sides, we get: $\frac{N}{N_o} = e^{-\lambda t}$

or $N = N_o e^{-\lambda t}$

This relation shows that the decay of nucleus is exponential with time.

Example 22.5. Find decay constant and activity of an element if its 100 out of 700 nuclei decay in 5 min.

Given: Original number of nuclei: 'N' = 700 Dacayed nuclei: 'ΔN' = 100

Time: 't' = 5 min = 300 s

To Find: Decay constant: λ = ? Activity: A = ?

Solution: To find decay constant we use: $\lambda = \frac{1}{N} \times \frac{\Delta N}{\Delta t}$

Putting values, we get: $\lambda = \frac{1}{700} \times \frac{100}{300} = 4.76 \times 10^{-4} s^{-1}$

Now to find activity, we use:

$$A = \lambda\, N$$

Putting values, we get: $A = 4.76 \times 10^{-4} s^{-1} \times 700 = 0.333 \text{ Bq} = 9 \times 10^{-12} \text{ Ci}$

Assignment 22.5

Find decay constant and activity of an element if its 240 out of 780 nuclei decay in 12 seconds.

22.4.2 Relation between Decay Constant and Half-Life

The relation between decay constant and half-life can be deduced by considering a radioactive element that has 'N_o' number of nuclei at a certain instant. If the half-life of the element is four hours, then after four hours the number of radioactive element's nuclei left will be '$N_o/2$'. After eight hours i.e. after two half-lives the number of nuclei of the original element left will be '$N_o/4$'' similarly after twelve hours i.e. after three half-lives the number of nuclei of the original element left will be '$N_o/8$'' and so on. To represent the variation of un-decayed nuclei as a function of time a graphical method is more suitable, as shown in Fig. 22.9.

From the graph it is very easy to find the half-life of a material. To derive the relation for half-life, we use equation:

$$N = N_o e^{-\lambda t}$$

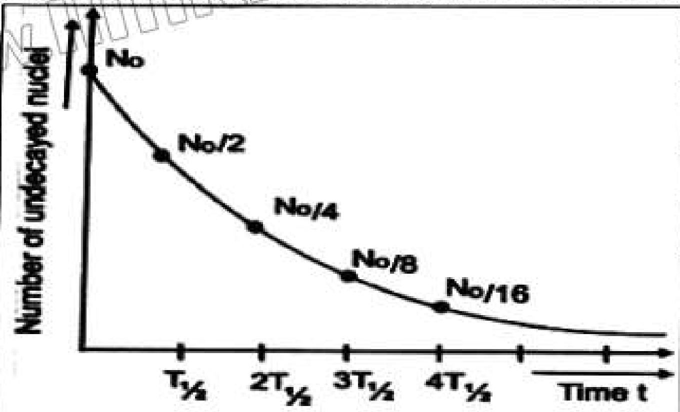
Now use the values for first half life of an element as: $N = \frac{N_o}{2}$ and $t = T_{1/2}$. Using these values in above equation we get:

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

or $$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking inverse of the above relation, we get:

$$2 = e^{\lambda T_{1/2}}$$



Time	N
0	10000
$T_{1/2}$	5000
$2T_{1/2}$	2500
$3T_{1/2}$	1250
$4T_{1/2}$	625

Figure 22.9: Graphical representation of nuclear decay

Now taking natural logarithm on both sides: $\ln 2 = \lambda T_{1/2}$

Now the relation of half-life of a radioactive element can be given as:

$$T_{1/2} = \frac{\ln 2}{\lambda} \quad \text{or} \quad T_{1/2} = \frac{0.693}{\lambda}$$

This equation shows the relationship between the decay constant and the half-life of an element. The half-life of a radioactive material is determined by measuring the activity of a given sample over a period of time. It is important to note that half-life is a characteristics property of each radioactive element and it is used to describe the rate at which a radioactive substance decays. The half-life of radioactive element is the duration it takes for half of the original amount of the substance to decay into a more stable form. The longer the half-life of a substance the slower is its rate of decay and less radiations it emits over time.

Example 22.6. Cesium-137 (a beta emitter used for treating cancer and in industrial devices to measure the thickness of materials) has a half-life of 9.5x10⁸ s. we have one mole of cesium at the start. What will be the decay constant and activity for this decay?

Given: Half-life of cesium-137 = $T_{1/2} = 9\times10^8$ s $\quad N_o = 6.02\times10^{23}$

To Find: Decay constant = λ = ? $\quad$ Activity = A = ?

Solution: The relationship between half-life and decay constant is given by:

$$T_{1/2} = \frac{0.693}{\lambda} \quad \text{or} \quad \lambda = \frac{0.693}{T_{1/2}}$$

Putting values, we get:

$$\lambda = \frac{0.693}{9.5\times10^8 s} = 7.3\times10^{-10} s^{-1}$$

Now to find the activity, we use: $A = \lambda N$

Putting values, we get: $A = 7.3 \times 10^{-10}\,s^{-1} \times 6.02 \times 10^{23} = 4.395 \times 10^{14}\,Bq = 1.188 \times 10^{4}\,Ci$

Assignment 22.6

The half-life of strontium (Sr-91) is 9.70 hours. Find its decay constant.

22.5 NUCLEAR REACTIONS

Nuclear reactions are processes in which one or more nuclides are produced from collision between two nuclei or one nucleus and a sub-atomic particle. A nuclear reaction is said to occur when an incident nucleus, particle or photon causes a change in the target nucleus.

Nuclear reactions are processes in which atomic nuclei interact, resulting in changes to the nucleus, such as fusion, fission, or radioactive decay, often releasing or absorbing energy.

A nuclear reaction changes the identity or characteristics of an atomic nucleus by bombarding it with an energetic particle. The bombarding particle may be an alpha particle, a gamma ray photon, a neutron, a proton or a heavy ion. These bombarding particles must have enough energy to approach the positively charged nucleus within the range of the strong nuclear force. Let the nucleus 'X' is bombarded by some light particle 'a' which products a nucleus 'Y' along with a light particle 'b' mathematically:

$$a + X \rightarrow Y + b$$

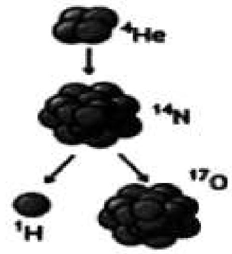


Figure 22.10: Nuclear reaction.

The first ever nuclear transmutation, as shown in Fig. 22.10, observed by Rutherford, is given below:

$${}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H + Q$$

Where Q is the energy released or absorbed equivalent to the difference of the rest masses of the elements on both sides of nuclear reaction. If 'Q' is positive the energy will be liberated in the reaction called exothermic reaction and if 'Q' is negative the energy is absorbed in the reaction called endothermic reaction. If thermal neutron (neutrons with energy equal to surrounding atoms i.e. about 0.025 eV) is bombarded on nitrogen-14 nucleus, the following reaction can be produced with release of a large amount of energy:

$${}^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}H + Q$$

The product in this case is the radio carbon-14, a proton and heat energy. Similarly, deuteron which has a unit positive charge and must have high energy to cause an induced nuclear reaction as:

$${}^{2}_{1}H + {}^{55}_{25}Mn \rightarrow {}^{56}_{26}Fe + {}^{1}_{0}n + Q$$

All the conservation laws must be followed in a nuclear reaction including law of conservation of mass-energy, momentum and charge etc. The main types of nuclear reactions due to low value of binding energy per nucleons are nuclear fusion and nuclear fission which are explained below.

22.5.1 Nuclear Fusion

Do you ever think the energy from the stars coming to us is obtained by which process? What is the fuel of the Sun and how it produces tremendous amount of energy? Answer to these questions lies in a simple nuclear reaction called the nuclear fusion reaction which can be stated as:

A process where two or more light atomic nuclei come close to each other to form a heavier nucleus by releasing a large amount of energy is called nuclear fusion.

Small nuclei combine to form heavier nuclei, releasing a large amount of energy in the form of heat. This process occurs naturally in stars where the high temperature and pressure causes the nuclei to collide and fuse. The Sun's core is the site of several nuclear reactions that powers its energy output. Scientists are currently working on developing nuclear fusion as a potential clean and sustainable source of energy. A simple fusion reaction is shown in Fig. 22.11 and can be written as:

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n + Q \quad \text{(22.1)}$$

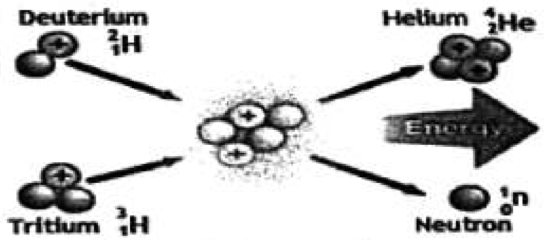


Figure 22.11: Fusion reaction.

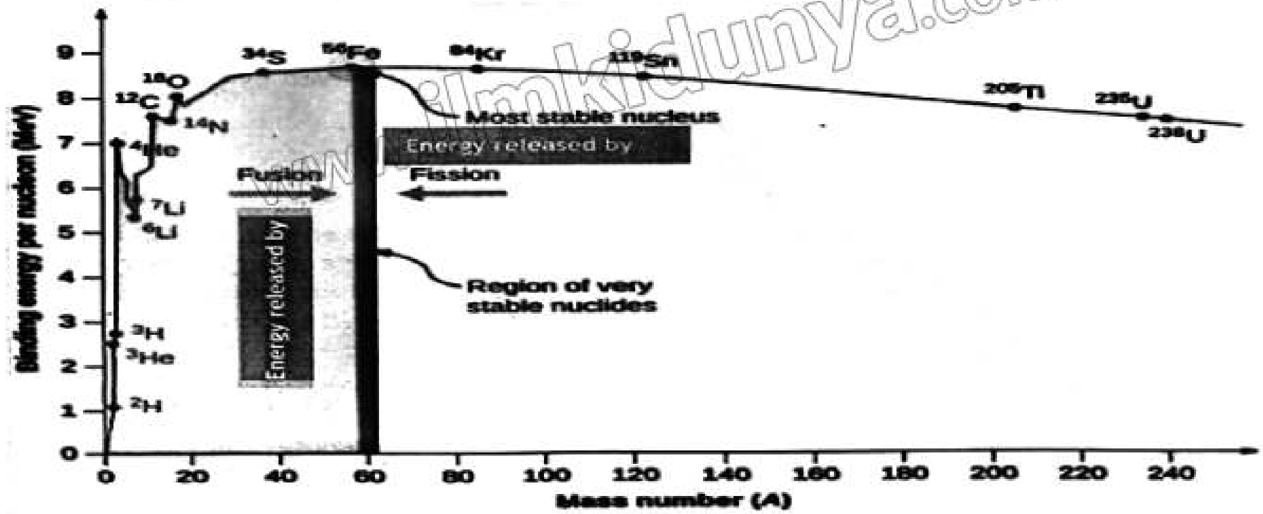


Figure 22.12: Fission and Fusion reaction with binding energy curve.

Why small nuclei tend to fuse with each other? This is due to their low binding energy per nucleon which makes them less stable as compared to the elements of region of greater stability. They tend to fuse with each other to get themselves more stable. As from Fig. 22.12 the binding energy per nucleon increases when the mass number increases and becomes maximum for '$^{56}_{26}Fe$' whose atomic mass is A = 56 which is 8.8 MeV per nucleon. The initial increase in binding energy per nucleon favors the fusion reaction.

Energy in Nuclear Fusion: We can calculate the amount of energy released during nuclear fusion reaction by using the concept of binding energy per nucleon as the total energy of fusing nuclei in equation (22.1) can be given:

Mass of Deutrium = 2.014102 u   Mass of Tritium = 3.016049 u

Total Mass = 5.030151 u

To covert this mass in kilograms, we proceed as:

$$\text{Total Mass} = 5.030151\ u \times 1.660 \times 10^{-27}\ kg/u = 8.350 \times 10^{-27}\ kg$$

Energy of L.H.S of equation (22.1) is:

$$E = \Delta m c^2 = 8.350 \times 10^{-27}\ kg \times (3 \times 10^8\ ms^{-1})^2 = 7.515 \times 10^{-10}\ J = 4696.87\ MeV \quad \text{(i)}$$

Now the total energy of product nuclei and particle in equation (22.1) can be given:

Mass of Helium = 4.002603 u   Mass of Neutron = 1.008665 u

Total Mass = 5.011268 u

To covert this mass in kilograms we get:

$$\text{Total Mass} = 5.011268\ u \times 1.660 \times 10^{-27}\ kg/u = 8.319 \times 10^{-27}\ kg$$

Energy of L.H.S of equation (22.1) is:

$$E = \Delta m c^2 = 8.319 \times 10^{-27}\ kg \times (3 \times 10^8\ ms^{-1})^2 = 7.4871 \times 10^{-10}\ J = 4679.438\ MeV \quad \text{(ii)}$$

Using these values in the equation (22.1), we get the amount of energy obtained/released in the above fusion reaction as:

$$4696.87\ MeV = 4679.438\ MeV + Q$$

or $$4696.87\ MeV - 4679.438\ MeV = Q$$

or $$Q = 17.43\ MeV$$

Here positive value of Q shows that it is an exothermic reaction and heat is released in this reaction. Hence a single event of fusion of deuterium and tritium forming helium and a neutron, releases 17.43 MeV energy.

22.5.3 Nuclear Fission

Otto Hahn and Fritz Strassmann while working upon the nuclear reaction find a startling discovery. They discovered a phenomenon called nuclear fission reaction which can be stated as:

A process where the nucleus of an atom splits into two or more smaller nuclei, with release of large amount of energy is called nuclear fission.

The *process is typically* initiated by bombarding the nucleus with a neutron, which causes it to become unstable and split apart. There are different possible nuclear reactions with different daughter nuclei they may be different nuclei or different isotopes of same nuclei. For example, a nuclear reaction is shown in Fig. 22.13 and can be given as:

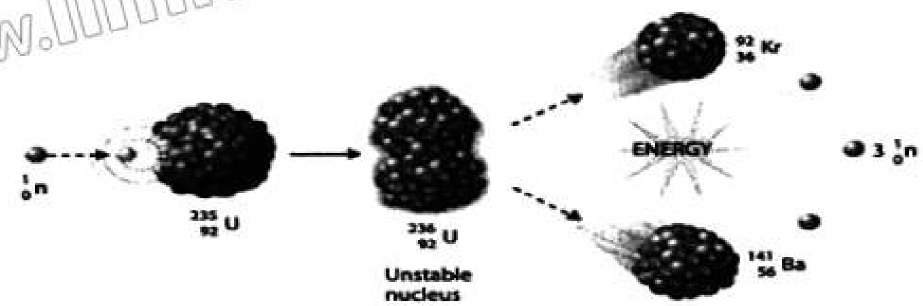


Figure 22.13: Fission reaction.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + Q \text{ \_\_\_\_\_ (i)}$$

In this reaction, when a slow neutron hits uranium-235, it forms an unstable nucleus (i.e., uranium-236 $^{236}_{92}U^{*}$) that breaks into barium, krypton, neutrons, and releases a lot of energy. Fission does not give the same product every time. Fission of uranium-235 can also yield different elements, such as shown in the following equation:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n + Q$$

The number of neutrons emitted during fission reaction is not necessarily to be three. There are some reactions which emit two neutrons per fission event. The product may be different isotopes of the same elements as in equation (i), which is shown in Fig. 22.14 and can be given as:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U^{*} \rightarrow ^{139}_{56}Ba + ^{95}_{36}Kr + 2\,^{1}_{0}n + Q$$

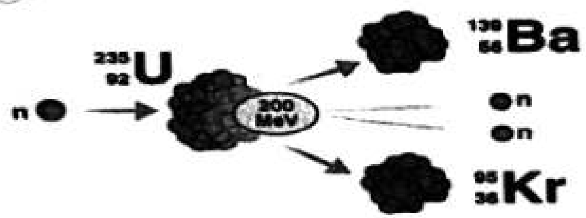


Figure 22.14: Fission reaction with different isotopes.

Nuclear fission has practical applications in nuclear power plants to generate electricity as well as in nuclear weapons. As from Fig. 22.12, we can see that the binding energy per nucleon is small for heavier nuclei and hence they are unstable and tend to get more stable condition. The fission reaction occurs due to instability of heavy nuclei such that they may get small fragments having larger stability.

Fission Chain Reaction: A single fission reaction may release two or three neutrons which can further cause fission reactions. That further reactions produce more neutrons that cause more fission reaction and so on.

A fission reaction in which every time at least one released neutron produces further fission reaction then such succession of fission reactions is called fission chain reaction.

The neutrons produced during fission reaction are fast neutrons and can escape out of the material without producing more fission reactions, as for fission, slow neutrons are required. To sustain the fission process, we need one neutron per fission to be capable of carrying out next fission. If the size of material is large, some of these neutrons may be captured to produce further fission. Hence, we need a minimum mass of material to sustain fission chain reaction which is called as critical mass and can be defined as:

The minimum mass of the material to sustain the fission chain reaction is known as the critical mass.

If the mass of the material is less than the critical mass the fission process would soon come to an end. But if the mass of the material is greater than the critical mass the fission process will get uncontrolled and would lead to cause the nuclear bomb. If the mass of the sample is equal to the critical mass (or the conditions are suitable for producing one more fission reaction per event) controlled nuclear energy will be produced as in case of nuclear reactors. This process is used in nuclear reactors and nuclear weapons. In a nuclear weapon, the nuclear fission is uncontrolled leading to a rapid and explosive release of energy. The potential for uncontrolled chain reactions is what makes nuclear weapons so dangerous and that's why strict safety protocols are to be followed to prevent any incident. In a nuclear reactor, the chain fission reaction is controlled using different techniques which enable us to produce energy. A fission chain reaction of uranium-235 is shown in Fig. 22.15. The volume occupied by the critical mass of a material is known as critical volume.

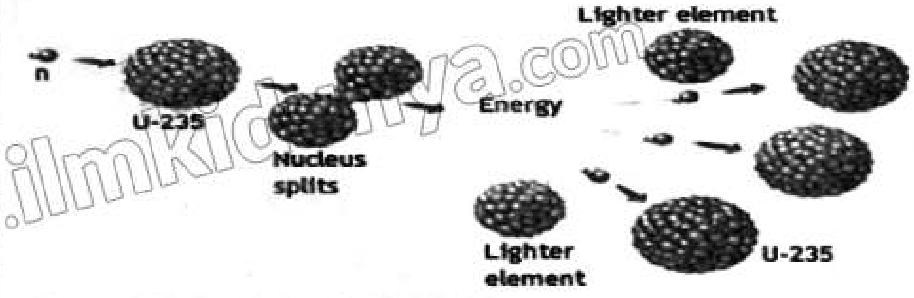


Figure 22.15: Fission chain reaction.

Energy in Fission Reaction: A huge amount of energy is released in nuclear reactions, particularly in nuclear fission reaction, which is much greater than the amount of energy released in a typical chemical reaction. For example, a single fission event like the splitting of uranium-235 nucleus releases about 200 MeV of energy. On the other hand, a chemical reaction like combustion of fossil fuels release energy on the order of a few electron volts (eV) per reaction. For example, energy released by one molecule of methane (CH_4) gas is about 9.2 eV.

The fission of 1 kg of uranium-235 can produce about 2.5 million times more energy than burning of 1 kg of coal. Hence, nuclear fission reaction releases energy that is millions of times greater than that of chemical reactions. Due to this huge energy difference nuclear power plants are so useful as compared to traditional chemical based energy sources.

Example 22.7. Find the amount of energy released by the following fission reaction if the masses of uranium, barium and krypton are 235.12142 u, 140.883 u and 91.9262 u, respectively.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + Q$$

Given: Mass of uranium-235: 235.12142 u   Mass of barium-141: 140.883 u
Mass of crypton-92: 91.9262 u   Mass of neutron: 1.008665 u

To Find: Energy: Q = ?

Solution: Total mass on L.H.S = 1.008665 u + 235.12142 u = 236.1301 u

So, Total energy on L.H.S = $236.1301\ u \times 931.5\ MeV/u\ c^2 = 219955.188\ MeV/c^2$

Now Total mass on R.H.S = $140.883\ u + 91.9262\ u + (3 \times 1.008665)\ u = 235.8352\ u$

So, Total energy on R.H.S = $235.8352\ u \times 931.5\ MeV/u\ c^2 = 219680.489\ MeV/c^2$

Now to find the energy 'Q' for the above fission, put values in reaction equation:

$$219955.188\ MeV/c^2 = 219680.489\ MeV/c^2 + Q$$

or

$$Q = 219955.188\ MeV/c^2 - 219680.489\ MeV/c^2$$

$$Q = 274.699\ MeV/c^2$$

Positive sign shows that the energy is released during the fission reaction.

Assignment 22.7

Find the amount of energy released by the following fission reaction if the masses of uranium, Xenon, Strontium and neutron are 235.12142 u, 139.9055 u, 93.9064 u and 1.008665 u respectively constant.

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n + Q$$

22.6 NUCLEAR REACTORS

Large amount of energy is released in a nuclear fission reaction; therefore, it is carried under controlled conditions. Then this energy is used for useful purposes like in the production of electricity and radioisotopes.

Nuclear reactor is a device used to initiate and control a nuclear fission chain reaction.

A nuclear reactor typically consists of the core, fuel rods, control rods or safety rods, moderator, coolant (heat exchanger) and shielding unit. A nuclear reactor is shown in Fig. 22.16.

Core of Reactor: The core is the main component of a nuclear reactor which contains the fuel in which the nuclear fission process takes place. The core is the heart of a reactor where the critical processes of nuclear fission and heat generation occur which enables the reactor to function as a source of energy. It is designed to sustain a controlled nuclear fission chain reaction. The main parts in the core of the nuclear reactor are:

Fuel Rods: The fuel used in nuclear reactor is typically Uranium-235 or Plutonium-239, which is in the form of small pallets and arranged into long rods of roughly 1 cm in diameter. Most reactors use uranium in which the amount of Uranium-235 is enriched to about 3%. These rods are placed into the reactor's core.

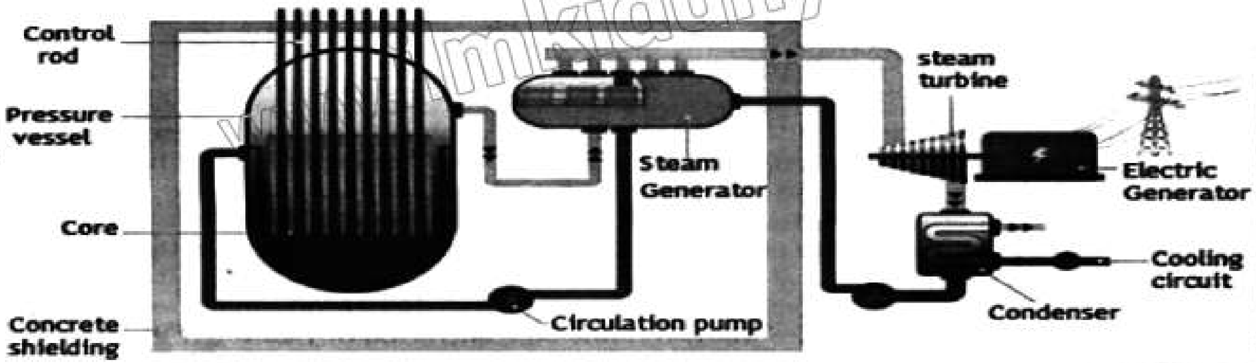


Figure 22.16: Schematic diagram of Nuclear Reactor.

Control Rods: Control rods also called as safety rods play very important role in the action of nuclear reactor as they are used to control the extent of reaction and do not allow the fuel to react at once. They are made of a material such as boron or cadmium that absorbs neutrons. These rods control the fission reaction by absorbing some of the neutrons and preventing them from causing additional fission reactions as for the output of a reactor to be constant only one neutron from each event should process further fission reaction. These rods can move into or out of the reactor's core as per need.

Moderator: Material that slows down the neutrons is called moderator. Neutrons with energies of about 0.025 eV are used in fission of uranium while the neutrons emitting from fission reaction are fast and have energies in the range of MeV which is not required for fission. The commonly used materials used as moderator are water and graphite that slows down the neutrons produced by fission as slowing down the neutrons make them more likely to cause additional fission.

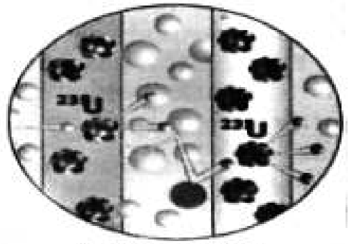


Figure 22.17: Moderator.

Coolant: The coolant or heat exchanger is a fluid such as water or helium that circulates through the reactor's core and carries away the heat produced by the fission process. This heat is used to produce steam which in turns derives the turbines to generate electricity. It also serves the purpose to cool the fuel rods and moderators from excess heat. In the absence of a coolant the core can melt under the enormous heat produced by the fission process. After passing through the generator the coolant passes through the condenser which cooled down the material before entering again into the reactor's core.

Shielding Unit: The by-product of a fission reaction includes strong radiations and energetic particles which are harmful to human and the atmosphere. To keep human being safe, we have to shield those radiations and high energy particles within the reactor such that they would not release into the atmosphere. For the above said purpose we use a thick layer of concrete or other material that surrounds the reactor vessel. It protects workers and the environment from radiations emitted by the reactor.

Enrichment of Uranium

The uranium found in nature has different isotopes and each isotope has different properties and characteristics. Every isotope of uranium is not equally favorable for the nuclear fission chain reaction. Uranium usually has two main isotopes i.e., Uranium-235 and Uranium-238. About 99.3% of natural uranium is U-238 but unluckily it is not directly fissile but it can capture neutrons and can be converted into plutonium-239 (Pu-239) which is fissile. On the other hand, uranium-235 is about 0.7% of natural uranium and this isotope is fissile means it can sustain a chain reaction by absorbing neutrons and undergoes fission. Hence uranium fuel needs to be enriched because natural uranium contains too low concentration of fissile U-235. Enrichment increases the proportion of U-235 allowing reactors to achieve and maintain criticality, operate efficiently, manage reactor design and fuel usage effectively.

The process of converting non fissile material like uranium-238 into a fissile material like uranium-235 is called enrichment.

To sustain a controlled nuclear chain reaction a reactor needs a sufficient concentration of fissile material i.e. U-235 in nature its concentration is as low as 0.7% which is not sufficient and nuclear reactor cannot be run on this percentage. Enriched uranium typically contains 3-5% U-235 due to this higher concentration of U-235 it improves the efficiency of reactor making it more feasible to maintain a steady and controlled reaction.

Various methods can be used to enrich uranium including gaseous diffusion method, as shown in Fig. 22.18, gas centrifugation and laser enrichment. The most common method is gas centrifugation which uses rapidly spinning centrifuges to separate uranium isotopes based on difference in their masses.

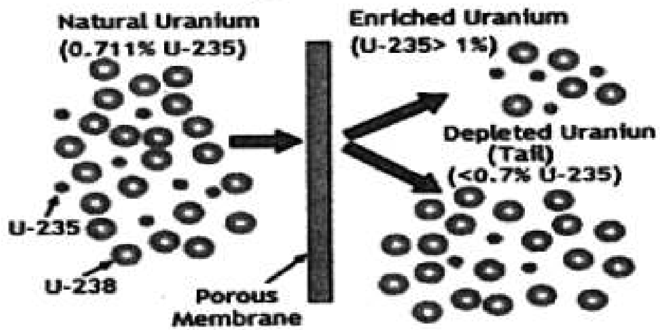


Figure 22.18: Gaseous diffusion uranium enrichment process.

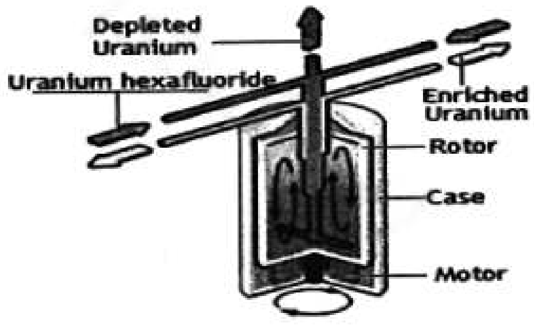


Figure 22.19: Gas Centrifuge.

The gas centrifugation process is shown in Fig. 22.19, in which a motor rotates the case which contained uranium hexafluoride. Gaseous diffusion enrichment is a method used to enrich uranium by converting it into a gas (Uranium hexafluoride) and passing it through porous barriers. The lighter U-235 molecules pass through more easily than the heavier U-238 molecules, gradually increasing the concentration of U-235. This process is repeated multiple times to achieve the desired level of enrichment.

22.7 ENERGY IN ANNIHILATION REACTIONS

Annihilation reactions are fundamental in particle physics and have practical applications in technologies like PET Scans in medical imaging. It occurs when a particle collides with its corresponding anti-particle resulting their mutual destruction and conversion of their mass into energy. This process is governed by the principles of conservation of momentum and mass-energy means total energy before and after the annihilation remains the same, as described by Einstein's equation $E = mc^2$. The total momentum also conserves in these reactions i.e. the total momentum of photons will be equal to the initial momentum of the particle and anti-particle. The most common example of annihilation reaction is electron-positron annihilation to produce two gamma ray photons and can be given as:

$$e^- + e^+ = \gamma + \gamma \qquad \text{\_\_\_\_\_\_ (22.2)}$$

Another annihilation reaction can be of proton and anti-proton in which they annihilate to produce photons or various other particles like pion and kaon, such reactions can be given as:

$$p^+ + p^- = \gamma + \gamma$$

or $$p^+ + p^- = \pi^+ + \pi^- + \pi^0$$

The annihilation reaction of proton and anti-proton is shown in Fig. 22.28.

Figure 22.19: Proton anti-proton annihilation.

The energy in electron-positron annihilation given by equation (22.2) can be calculated as:

The rest mass energy of an electron can be found by using equation $E = m_o c^2$ while the rest mass of an electron and positron is $9.1 \times 10^{-31}\,kg$. Now to calculate the rest mass energy of the electron and positron we can use:

$$E = m_o c^2 + m_o c^2 = 2hf$$

$$E = m_o c^2 = 9.1 \times 10^{-31}\,kg \times (3 \times 10^8\,ms^{-1})^2$$

or $$E = m_o c^2 = 8.19 \times 10^{-14}\,J$$

This energy can be converted into electron-volt units as:

$$E = \frac{8.19 \times 10^{-14}}{1.6 \times 10^{-19}}\,eV = 511875\,eV$$

As positron has the same mass as that of electron hence it also has the same energy so total energy of the annihilating particles can be given as:

$$E = 2 \times 511875\,eV = 1.02\,MeV$$

This is minimum amount of energy produced by the annihilation reaction. Hence each gamma ray photon carries energy of 0.51 MeV. For the case when electron and positron are moving then their kinetic energy is also included in the total energy which determines the energy of the emitted gamma ray photons and can be written as:

$$K.E_{electron} + K.E_{positron} + 2m_oc^2 = 2hf \quad \_\_\_\_\_\_ (22.3)$$

Example 22.8. Find the amount of energy released by the annihilation of electron and positron initially moving with kinetic energy of 0.12 MeV each, while the rest mass energy of electron and positron is 0.51 MeV.

Given: Rest mass energy of electron: $m_oc^2 = 0.51$ MeV

Rest mass energy of positron: $m_oc^2 = 0.51$ MeV

$K.E_{electron} = 0.12$ MeV $\quad K.E_{positron} = 0.12$ MeV

To Find: Energy of gamma ray photon: $hf = ?$

Solution: To find energy we use equation 22.15:

$$K.E_{electron} + K.E_{positron} + 2m_oc^2 = 2hf$$

Putting values, we get: $0.12 \text{ MeV} + 0.12 \text{ MeV} + 2\times(0.51 \text{ MeV}) = 2hf$

or $1.26 \text{ MeV} = 2hf$

or $hf = 0.63 \text{ MeV}$

Hence the energy of each gamma ray photon will be 0.63 MeV.

Assignment: 22.8

Find the energy of gamma rays photon released by the proton and anti-proton annihilation reaction where the mass of proton and anti-proton is 1.007276 u. The reaction can be given as: $p^+ + p^- = \gamma + \gamma$

22.8 MEDICAL USES OF RADIATIONS

Radiations have vast medical treatment and diagnostics applications. The most commonly used radiations you would ever use are the medical x-rays, gamma ray therapy and other radiations. These are widely used for different purposes in medical science. Medical tracer and PET scanner are some of the techniques commonly used in medical diagnostics.

22.8.1 Medical Tracer

A medical tracer is a substance which contains some radioactive nuclei. It is used in medical imaging to study the functions of tissues and organs. When we introduce this substance into the body the tracer is absorbed by the specific tissue being examined, as shown in Fig. 22.20. The radioactive nuclei emit gamma rays which can be detected by imaging equipment such as positron emission tomography (PET) or single photon emission computed tomography

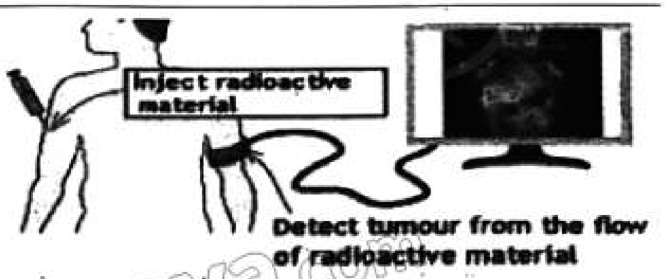


Figure 22.20: Medical Tracer.

(SPECT) scanners. These images help doctors diagnose and monitor various conditions such as cancer, heart disease and neurological disorders by providing detailed information about the processes occurring within the body. The tracer is usually introduced into the body through injection into the blood stream, ingestion or inhalation depending upon the type of study and the area of interest. When tracer enters into the body it is absorbed by specific tissues due to its affinity for certain biological processes. Tracers are valuable in research settings to study how diseases develop and progress as well as to explore new treatments and diagnostic methods. There are many radioactive materials which are used as medical tracers depending upon the need of study and their properties required. Some of the commonly used materials as medical tracers and their use are given in the table below:

Table 22.4: Medical tracers and their use

Element	Use	Half-life
Technetium-99	Bone scan, cardiac stress and thyroid	6 hours
Fluorodeoxyglucose	Metabolic activity and cancerous tumors	110 minutes
Iodine-131	Therapeutic applications, hyperthyroidism and thyroid cancer	8 days
Iodine-123	Thyroid gland, brain and heart scan	13 hours
Gallium-67	Infections, tumors, lymphoma and lung cancer	78 hours
Thallium-201	Myocardial perfusion imaging, heart function and coronary artery disease	73 hours
Radium-223	Bone metastases, prostate cancer and reduce cancer spread in bone tissues	11.4 days
Carbon-11	Neurodegenerative diseases and certain types of cancer	20 minutes
Oxygen-15	Measure blood flow and metabolism in tissues	2 minutes

22.8.2 PET Scanner

Positron emission tomography (PET) is a powerful imaging technique that allows us for detailed observation of metabolic physiological processes within the body. For diagnostic under PET scanner, we perform the following steps:

i) Inject the patient with some positron emitting radionuclide such as carbon-11. The tracer travels through the blood stream and accumulates in tissues or organs which are under studies.

ii) The radionuclide within the tracer undergoes radioactive decay emitting a positron. A positron is an anti-particle of electron which has the same mass as that of electron but of opposite polarity of charge.

iii) The emitted positron from C-11 travel a short distance and they encounter electrons in the surrounding tissues.

iv) When the electron and positron meet, they annihilate each other producing gamma ray photons, as shown in Fig. 22.21.

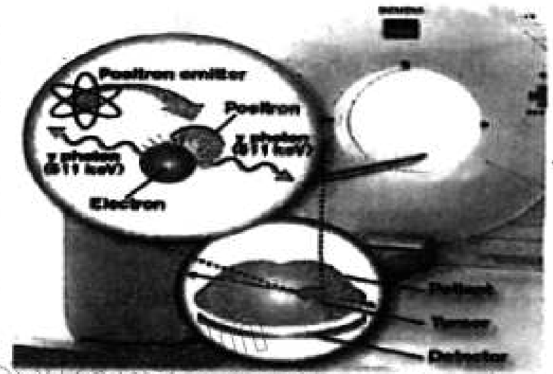


Figure 22.21: Annihilation within body.

v) The two gamma ray photons result from electron positron annihilation emitted with energy of 511 keV each and travel in opposite direction to conserve momentum.

vi) PET scanner consists of a ring of detectors that encircle the patient, as shown in Fig. 22.22. These detectors are sensitive for gamma ray detection when the gamma rays strike the detectors they are converted into an electric signal. In this process these gamma ray photons escape the body and travel outside of the body towards the ring detector that surrounds the body.

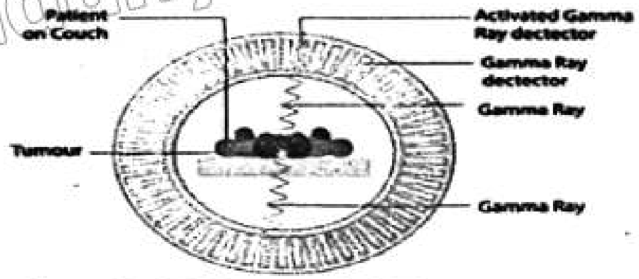


Figure 22.22: Detectors in PET.

vii) The scanner detects simultaneous arrival of many pairs of gamma rays travelling at 180°. Then it records their time of arrival and their spatial information. The detected signals are then processed by a computer to determine the location of annihilation event within the body. The computer then makes an image from the detected photon pair providing insight into the body.

The complete working diagram of positron emission tomography (PET) scanner is shown in Fig. 22.23.

Figure 22.23: Schematic diagram of PET scanner.

SUMMARY

- **Nuclear physics:** Nuclear physics is a branch of physics that deals with the study of the atomic nucleus, its properties and interactions.
- **Mass defect:** The difference between the mass of the nucleus and sum of the masses of its constituent particles is called as mass defect.
- **Binding energy:** The minimum energy required to break an isolated nuclei into its constituent particles is called the binding energy.
- **Binding energy per nucleon:** It is the total binding energy of an atom divided by the number of nucleons.
- **Radioactivity:** The natural process of emission of radiations from unstable nuclei is called radioactivity.
- **Alpha decay:** Alpha decay occurs when an atomic nucleus emits an alpha particle, which is a helium nucleus consisting of two protons and two neutrons.

- **Beta decay:** Beta decay occurs when an atomic nucleus emits a beta particle, which is an electron or a positron.
- **Gamma decay:** Gamma decay occurs when an atomic nucleus transitions from a higher energy state to a lower energy state, emitting a gamma ray in the process.
- **Half-life:** The time it takes for half of the radioactive nuclei to decay is called as half-life of that element.
- **Curie:** When there is an activity at the rate of 3.70×10^{10} decays per second then the activity is equal to one curie.
- **Fusion:** A process where two or more light atomic nuclei come close to each other to form a heavier nucleus by releasing a large amount of energy is called nuclear fusion.
- **Fission:** A process where the nucleus of an atom splits into two or more smaller nuclei, some particles like neutron and a large amount of energy is called nuclear fission.
- **Fission chain reaction:** A fission reaction in which every time at least one released neutron goes further fission then such succession of fission reactions is called fission chain reaction.
- **Critical Mass:** The minimum mass of the material to sustain the fission chain reaction is called as the critical mass.
- **Critical volume:** The volume occupied by the critical mass of a material is known as critical volume.
- **Nuclear reactor:** Nuclear reactor is a device used to initiate and control a nuclear fission chain reaction.
- **Control rods:** Control rods are used to control the extent of reaction and do not allow the fuel to react at once.
- **Moderator:** Material that slows down the neutrons is called moderator.
- **Enrichment of uranium:** The process of converting non fissile material like uranium-238 into a fissile material like uranium-235 is called enrichment.
- **Gas diffusion method:** Gas diffusion method is used for enrichment of uranium in which the gas is allowed to diffuse from a porous wall.
- **Gas centrifuge method:** The most common method is gas centrifugation which uses rapidly spinning centrifuges to separate uranium isotopes based on difference in their masses.
- **Medical tracer:** A medical tracer is a substance which contains some radioactive nuclei.
- **PET scanner:** Positron emission tomography (PET) is a powerful imaging technique that allows us for detailed observation of metabolic physiological processes within the body.

Formula Sheet

$\Delta m = [(A - Z)\, m_n + Z\, m_p] - m_{nucleus}$ $\qquad E = \Delta m\, c^2$ $\qquad \frac{\Delta N}{\Delta t} = -\lambda N$

$f = \frac{E}{A} = \frac{((A - Z)\, m_n + Z\, m_p - m_{nucleus})\, c^2}{A}$ $\qquad A = \lambda N$ $\qquad T_{1/2} = \frac{0.693}{\lambda}$ $\qquad N = N_o e^{-\lambda t}$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) The process by which a heavy nucleus splits into two smaller nuclei is called:
   A. Fission B. Fusion C. Alpha decay D. Beta decay
2) Radiation with highest ionizing power is:
   A. Gamma rays B. X-rays C. Alpha particle D. Beta particle
3) Which of the following particles has almost the same mass as a proton but carries no charge:
   A. Neutron B. Proton C. Electron D. Beta particle
4) The charge on an alpha particle is:
   A. −1 B. −2 C. +1 D. +2
5) In the nucleus of uranium-235, the number of neutrons is:
   A. 92 B. 143 C. 235 D. 134
6) The half-life of radium is 1590 years. In how many years shall the Earth loss all its radium due to radioactive decay?
   A. 795 B. 1590 C. 3180 D. Infinite
7) The energy released in nuclear reactor is produced by:
   A. Fission B. Fusion C. Coal D. Gas
8) C-14 has a half-life 5730 years. The number of nuclei in a sample will drop to 1/8 of initial quantity in_________ years:
   A. 1.44×10^4 B. 1.72×10^4 C. 2.58×10^4 D. 2.85×10^4

Short Questions

1) What are some of the potential benefits and drawbacks of using nuclear energy as a source of electricity compared to other forms of energy?
2) What happens to the atomic number and mass number of nucleus that (a) emits electron (b) undergoes electron capture (c) emits α particle?
3) Why does the low energy alpha particle not make physical contact with the nucleus when an alpha particle is headed directly towards the nucleus of an atom?
4) An alpha particle has twice the charge of a beta particle. Why does the former deflect less than the later when passing between electrically charged plates, assuming they both have the same speed?
5) Why uranium fuel needs to be enriched before use?

6) If U-238 undergoes alpha decay, what is the resulting nucleus?

7) How do chain reactions occur in nuclear fission?

8) How does the molecular weight difference between U-235 and U-238 hexafluoride molecules enable enrichment? What role do porous barriers play in the gaseous diffusion process?

Comprehensive Questions

1) Explain the terms mass defect and binding energy.

2) What is radioactivity? Explain.

3) How can you find energy from a nuclear decay?

4) Describe the importance of radiations in medical field?

5) What is nuclear fusion? Explain in detail. Also discuss some practical applications of this reaction.

6) What is nuclear fission? Explain in detail.

7) Discuss the function of the principle components of water moderated power reactor: such as: core, fuel, rods, moderator, control rods, heat exchange, safety rods and shielding.

8) Explain the exponential nature of radioactive decay.

9) Explain annihilation reaction in detail.

10) Illustrate how PET scanning works.

Numerical Problems

1) If an isotope has a half-life of 10 days and there are initially 5000 nuclei, how many will remain after 30 days? **(Ans: 625)**

2) If a nucleus of carbon-14 has a decay constant of 0.00012 s^{-1}, what is its half-life?

(Ans: 5775 s)

3) What is energy released when a nucleus of U-235 undergoes fission and release two neutrons?

(Ans: 1.793×10^{-11} J)

4) Calculate the number of nuclei of C-14 remain un-decay after 40110 years if the initial atoms were 10,000 (The half-life of C-14 is approximately 5,730 years). **(Ans: 4670)**

5) Calculate the mass defect and binding energy per nucleon for silver (Ag) nucleus if it's atomic mass in 107.905949 u. **(Ans: 41.031×10^{-12} J/nucleon)**

6) Calculate energy released in the following reaction:

$${}^{1}_{0}n + {}^{238}_{92}U \rightarrow {}^{96}_{38}Sr + {}^{140}_{54}Xe + 3{}^{1}_{0}n + Q$$

(Ans: -6.474×10^{-11} J)